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Demonstration on the nonlinear scattering of crossed ultrasonic beams in presence of single bubble in water

--Manuscript Draft--

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Abstract:	An apparatus consisting of an open thin-wall clear acrylic rectangular tank is used to resonate the volume cavity of water in an nx=1, ny=1, nz=3 mode; such that a mm sized bubble or less can be levitated in the center of the tank. The authors will utilize the experimental arrangement suggested by T.J. Asaki, P.L. Marston, and E.H. Trinh [“Shape oscillations of bubbles in water driven by modulated ultrasonic radiation pressure: Observations and detection with scattered laser light,” JASA 93, p 706-713, (1993)]. In which, their experimental apparatus involves a description of their piezoelectric driver, which couples into the bottom of the tank. Next, the demonstration consists of the nonlinear scattering of crossed ultrasonic beams of primary frequency components: f1=1.9 MHz, f2= 2.1 MHz; which, interact nonlinearly with the bubble to produce nonlinear scattering outside the interaction region at the combination frequency f+ = 4.0 MHz. The receiving 4 MHz transducer unit will measure the nonlinear scattering in the forward direction. Here the receiver is located in the plane formed by the axes of the primary acoustic beams at the angle of + or – 45 degrees with respect to each transducer axis.
Section/Category:	Physical Acoustics

1. INTRODUCTION

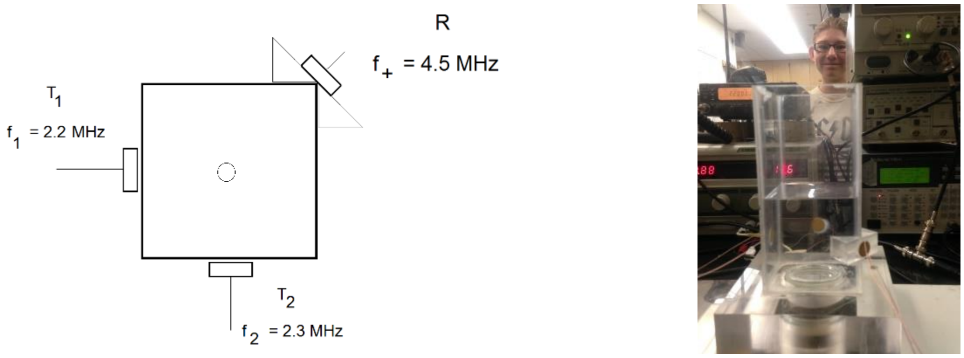

Figure 1. Left to right: Top view of non-linear scattering setup of crossed ultrasonic beams in the presence of a levitated air bubble in water; combined acoustic cell levitation apparatus with crossed beam transmitting and receiving transducers, and co-author KAH.

The project goal was to perform a non-linear scattering demonstration experiment using two mutually perpendicular crossed ultrasonic beams (frequency components $f_1 \sim 2.2$ MHz, $f_2 \sim 2.3$ MHz), which interact in the presence of a small air bubble in water. External to the interaction region, a sum frequency component, $(f_+ = f_1 + f_2 \sim 4.5\ MHz)$ was measured and compared to results obtained in the absence of the bubble. The sum frequency in the absence of the bubble might be due to side lobe primary wave components scattering non-linearly off the face of the receiving transducer. Other non-linear effects are possible due to the pressure release boundary conditions (for all walls) in the open rectangular tank.

In Fig. 1 above, the non-linear scattering geometry is presented with a photograph of the laboratory set-up. A challenge confronting the authors was to levitate an air bubble in water, such that the bubble would maintain its location in the interaction region (center) of the acrylic tank. The open rectangular (thin wall acrylic) tank apparatus is sometimes referred to as an acoustic levitation cell. Here; a capped, cylindrical shell, PZT-4 transducer (supported by an O-ring) located near the bottom of the cell, is used to generate a finite-amplitude acoustic pressure standing wave. This will generate an acoustic radiation force needed to counteract the buoyancy force on the bubble. It is known that a bubble which is larger than the resonant frequency size will levitate slightly above a pressure node, (our case of interest) while a bubble that is smaller than the resonant frequency size will levitate near a pressure anti-node. [1,2,3,4]

Given the Minnaert[5] resonant frequency of an oscillating air bubble in water, $f_0 = \sqrt{(3\gamma P_0/\rho_0)}\,/\,(2\pi R_0)$, where $\gamma = 1.4$, $P_0 \approx 1 \times 10^5 Pa$, $\rho_0 = 1000\ kg/m^3$, $f_0 = 26.8\ kHz$ the bubble radius will be $R_0 = 0.12\ mm$. In our experiment we will attempt to levitate a bubble roughly 1mm in diameter; therefore, we expect to observe the levitated vertical bubble position slightly above the pressure node (along the z-axis). A description of the levitation cell and cylindrical transducer will be presented later in this paper.

2. EXPERIMENTAL SETUP AND RESULTS

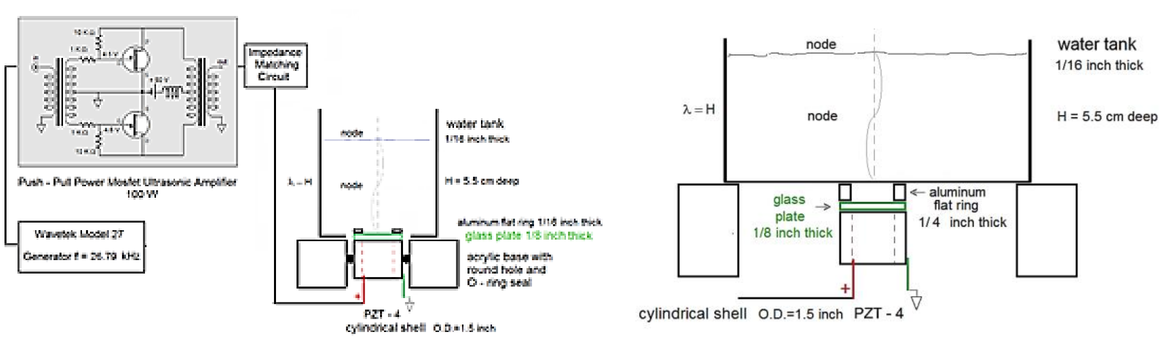

Figure 2. Supporting electronics with levitation cell #1 (tank base 5.7 x 5.7 cm²) and cell #2 (12.5 x 12. cm²).

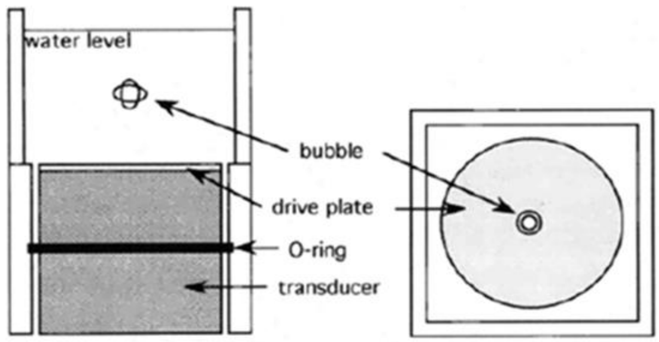

Figure 3. From Ref. 3, diagram of levitator in side and top views showing mounted transducer and water filled chamber. A bubble is illustrated at the approximate levitation position undergoing axisymmetric quadruple shape oscillations. The base of the open tank is 3 in. x 3 in. and the transducer height is also 3 in.

Using a coordinate system at the bottom south-west corner of the levitation cell – with pressure release surfaces on all sidewalls, an acoustic pressure mode is given by

$$p(x,y,z,t) = A_{n_x,n_y,n_z}\, sin(k_x x)\, sin(k_y x)\, sin(k_z x)\cos(\omega_{n_x,n_y,n_z}\, t + \phi_{n_x,n_y,n_z}) \qquad (1)$$

where the angular resonant frequency is given by

$$\omega_{n_x,n_y,n_z} = c\sqrt{\left({k_x}^2 + {k_y}^2 + {k_z}^2\right)} = c\sqrt{\left(\frac{n_x\pi}{L_x}\right)^2 + \left(\frac{n_y\pi}{L_y}\right)^2 + \left(\frac{n_z\pi}{L_z}\right)^2} \qquad (2)$$

Our radiation force on the bubble involves a standing wave pressure field with an n_x=0, n_y=0, n_z=2 mode shape.

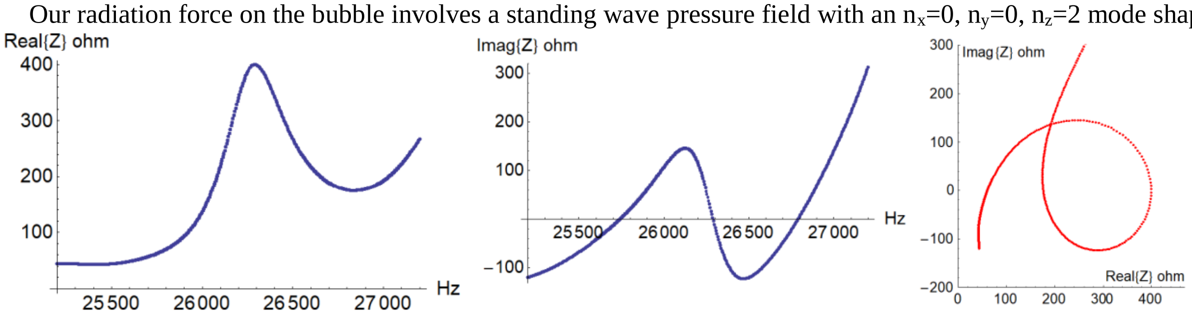

Figure 4. Left to Right: The real part and imaginary part of the electrical impedance Z of the cylindrical shell transducer vs. frequency; the circular impedance plot of the transducer (real {Z} vs. imaginary {Z}).

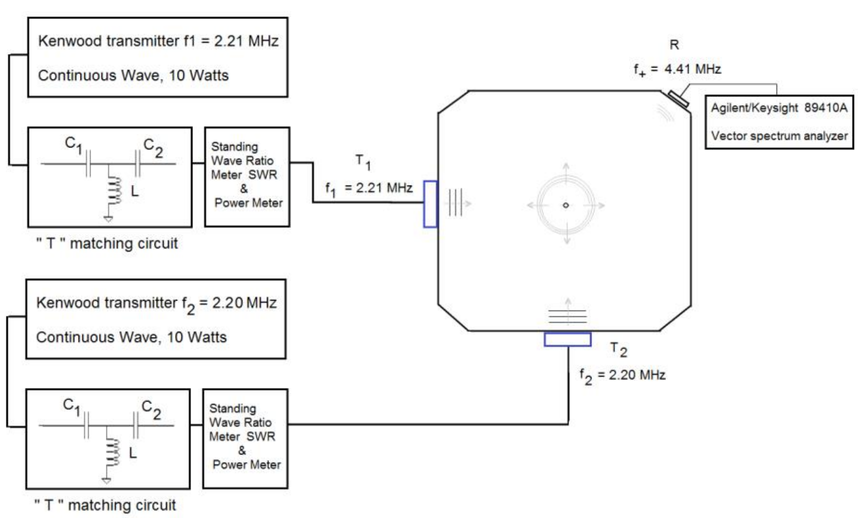

Figure 5. (On left): Schematic of cell #2 with supporting electronics and tank set-up for the nonlinear ultrasonic crossed beam scattering experiment in the presence of a single levitated bubble – detection at the sum frequency component 4.41 MHz.

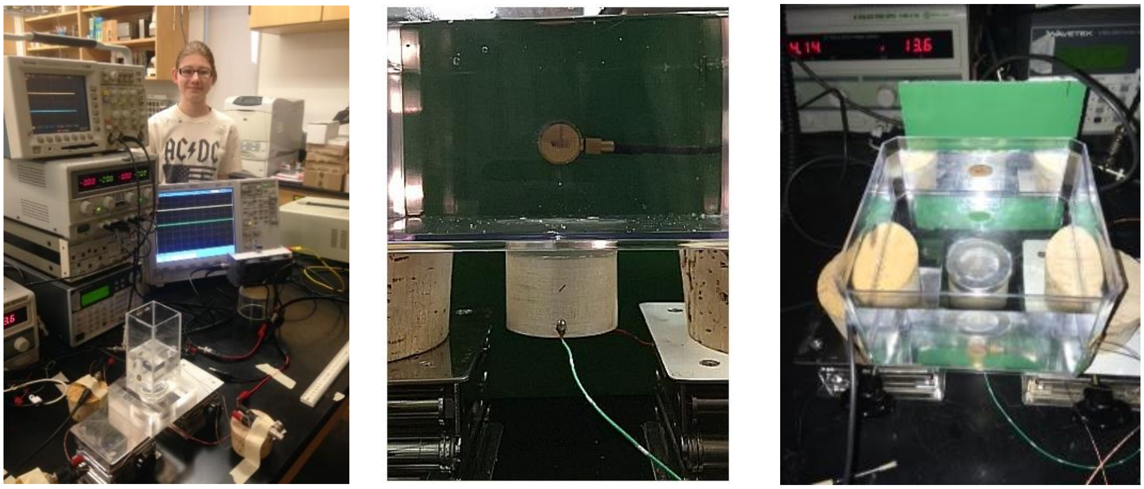

Figure 6. From left to right: Complete apparatus with levitation cell #1 and co-author KAH. Levitating bubble in tank appearing under transmitting transducer T_1 in center of tank (due to side-view camera angle). Overview of levitation cell #2 looking downward with transmitting transducers T_1 on north and T_2 west tank walls, and the receiver on the southeast corner.

The experimental setup using cell #2 is shown in Fig. 5 and 6 above. Here, the base dimensions of the tank are 12.3 by 12.3 by 5.5 cm. in depth. This larger cell allows for a reasonable separation between the transmitting and receiving transducer. The water height of 5.5 cm. is determined by having the vertical height be 1 wavelength at 26.8 kHz, using c=1482 m/s for the speed of sound in water. Fig. 7 below shows the receiving transducers spectrum in a 20 kHz band around the 4.14 MHz sum frequency. In the absence and presence of a single bubble (diam. ~ 1mm).

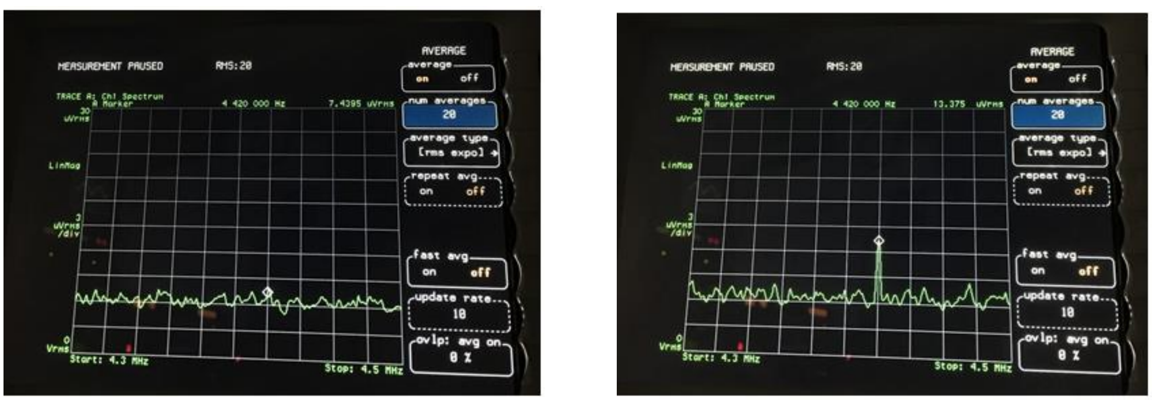

Figure 7. Received sum frequency component in the absence of a bubble in the interaction region. Received sum frequency component in the presence of a bubble in the interaction region. The transmitted primary frequency components are 2.21 and 2.20 MHz. The scattered sum frequency component is at $f_+ = f_1 + f_2 = 4.41$ MHz. Horiz. Scale is 2 kHz/Div. Vert. scale is 3 micro Vrms/Div.

3. HISTORIC DISCUSSION OF SCATTERING OF SOUND BY SOUND

The scattering of sound by sound had its origins in an experimental paper by Ingard and Pridmore-Brown[6]. The intricacies of the problem led to a spirited discussion involving the mechanisms which caused sum and difference frequencies to radiate outside the interaction region. Westervelt's theories[8] suggested that there was no scattering outside of the interaction region when the sources were plane waves; however, sound sources are a superposition of spherical waves, complicating any theoretical formulation. The evolution of the controversial problem has been sagaciously captured in Beyer's textbook[9]. This summary along with Refs. 10-12 will help the reader understand the full complexity of the problem.

4. CONCLUSION

The two major goals of the demonstration research project were (1) to levitate a bubble using acoustic radiation pressure in a so-called levitation cell, and (2) perform a non-linear scattering demonstration experiment using two mutually perpendicular crossed ultrasonic beams (primary frequency components f_1 ~ 2.21 MHz, f_2 ~ 2.20 MHz and nonlinearly generated combination sum frequency component f_+ ~ 4.41 MHz, which interact in the presence of a small air bubble in water. Both of these experimental projects were successful, however there is still much to be done in the future. First, we would like to to lower the frequency of the levitation cylindrical shell transducer, in order to increase the tank depth. Next, we would like to increase the base area of the tank, to allow the nonlinear scattering component of the experiment to be considered more of a free-field experiment than a wave fields subjected to pressure release boundary conditions. Further, higher frequency transducers (both transmitters and receiver) would make possible to have a narrower beam-width in the interaction region, and more absorption to eliminate multiple reflections. A future theoretical challenge would be to apply a resonant frequency bubble scattering theory, much like that in Ref 13 and 14 to our scattering geometry.

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