TOPIC: Formula for circles on the coordinate plane

Khan Academy Links:

Standard Form: https://www.khanacademy.org/math/geometry-home/analytic-geometry-topic/equation-of-a-circle/v/radius-and-center-for-a-circle-equation-in-standard-form

Completing the Square: https://www.khanacademy.org/math/geometry-home/analytic-geometry-topic/equation-of-a-circle/v/completing-the-square-to-write-equation-in-standard-form-of-a-circle

Standard Form

Standard Form for circles on the coordinate plane: $(x - h)^2 + (y - k)^2 = r^2$ where (h, k) is the center and r is the radius of the circle.

Example: A circle with equation of $(x - 3)^2 + (y + 4)^2 = 16$ has a center at $(3, -4)$ and a radius of 4. See graph to the right.

r =4
(3,-4)
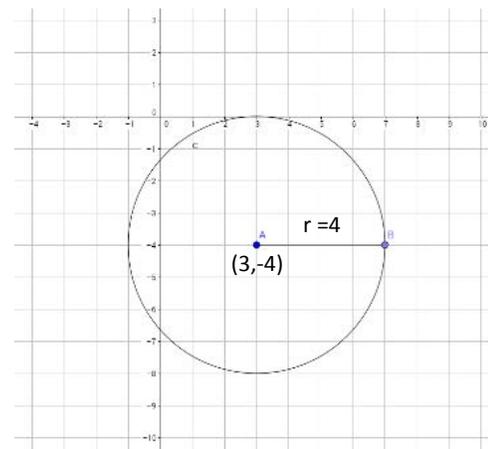
Completing the Square

Sometimes equations of circles are not in standard form, so it is almost impossible to find the center and/or radius. To get your equation into standard form, you will need to complete the square.

Example: Find the center of a circle with equation: $x^2 + 12x + y^2 - 4x + 15 = 0$.

Step 1: Move any constants to the right side of the equation.

Subtract 15 from both sides to get: $x^2 + 12x + y^2 - 4x = -15$

Step 2: Leave space after the x terms, after the y terms and two spaces after the constant. You are going to add the same thing to both sides keeping the equation balanced.

$$x^2 + 12x + \boxed{} + y^2 - 4x + \boxed{} = -15 + \boxed{} + \boxed{}$$

Step 3: Take the coefficient of the x term, 12. Half, then square that number. $\left(\frac{12}{2}\right)^2$ is 36.

Do the same for the y term, -4. Half, then square. $\left(\frac{-4}{2}\right)^2$ is 4. Your answer for this step will always be positive since you are squaring.

Now you will add your new numbers to their respective blanks making sure to also add to the right side of the equation to keep it balanced.

$$x^2 + 12x + 36 + y^2 - 4x + 4 = -15 + 36 + 4$$

Step 4: Remember that our goal is to get this mess to look like $(x - h)^2 + (y - k)^2 = r^2$. So we need to group our two trinomials and factor into binomial squares.

$$(x^2 + 12x + 36) + (y^2 - 4x + 4) = -15 + 36 + 4$$

The constant in the binomial will always be half of the coefficient of your x and y term, respectively. Factoring the x trinomial will give you $(x + 6)^2$ and factoring the y trinomial will give you $(y - 2)^2$.

$$(x + 6)^2 + (y - 2)^2 = -15 + 36 + 4$$

Step 5: Combine like terms on the right side of the equation. This could have been done earlier.

$$(x + 6)^2 + (y - 2)^2 = 25$$

Step 6: Identify the center and radius from the equations in standard form.

Center $(-6,2)$ and radius is 5. Remember that the coordinates are always the opposite sign of what is in the parentheses, and you must take the square root of the term on the right because standard form is $(x - h)^2 + (y - k)^2 = r^2$.

Sample Problems:

1. Identify the center of the circle with equation: $(x + 3)^2 + (y + 5)^2 = 36$.

2. What are the coordinates of the center of the circle with the equation

$$x^2 - 2x + y^2 + 6x + 6 = 0$$